Part 2 - INTEREST RATE PARITY and COVERED INTEREST ARBITRAGE

Exchange rates are not only important for considering the trade of goods and services, but also when investors are moving their money from one country to another for the purposes of financial investment.

In addition most major industrialized countries have floating exchange rates, meaning the relative values of currencies fluctuates continuously throughout the trading day.

Here's an example of a typical problem:

You want to decide whether to invest \$1 million of your money in Canada or France

Suppose the domestic interest rate in Canada is $\mathbf{r_D = 3\%}$
the interest rate in France is $\mathbf{r_F = 5\%}$

The spot exchange rate $\mathbf{p_0 = \frac{C\$}{€} = \$1.3021}$ or $\frac{1}{p_0} = \frac{€}{C\$} = €0.76799$

The future exchange rate $\mathbf{p_1}$ in $\frac{C\$}{€}$ terms = unknown !

Investing in France requires converting our dollar to €, and then converting the investment return back to dollars at the end of the year. This entails exchange rate risk because the spot price of the Euro a year from now is uncertain.

First you have to calculate the future exchange rate ($\mathbf{p_1}$) that would make an investor indifferent between investing in France or Canada.

Assuming investment of \$1 million:

Return 1 year from now from Canadian deposit: \$1 million (1+.03) = C\$1,030,000

Return 1 year from now from French deposit: \$1million (.76799)(1+.05) = **€**806,390

To be indifferent between either investment \$1,030,000= €806,390
you need the future exchange rate ($\mathbf{p_1}$) to be = $\frac{C\$}{€} = \frac{1,030,000}{806,390} = 1.2773$

The € has to depreciate from C\$1.3021 to C\$1.2773 to break even.
This represents a depreciation of the Euro of about 2% which makes sense because the interest rate is 2% higher in France than in Canada.
If you think the Euro will depreciate by more than 2% you would invest in Canada and borrow in France.
If you think the Euro will depreciate by less than 2%, or even appreciate, you would invest in France and borrow in Canada.

INTEREST RATE PARITY

is the relationship between the spot FX rate, the interest rate differentials and the forward rate. If interest parity holds, then an investor will realize the same return in domestic currency, regardless of which country the investment was placed in.
It implies that interest rate differentials should be an unbiased forecast of future exchange rate changes. (works imperfectly)

- $\mathbf{p_0}$ spot exchange rate which is units of domestic currency per 1 unit of FX
- $\mathbf{p_1}$ t-period ***UNKNOWN*** forward rate, or the price of FX t periods from now
- $\mathbf{r_D}$ interest rate on an investment maturing at time t in home country
- $\mathbf{r_F}$ interest rate on an investment maturing at time t in foreign country

Uncovered Interest rate parity equation (p₁ unknown): $(1+r_D) = (1+r_F)(1/p_0)Ep_1$

The word "uncovered" stems from the fact that we do not know the future exchange rate (p_1) with certainty and this leads us to a speculative strategy. **Expected future FX rate is $\underline{E}p_1$**

Equivalently, this ***arbitrage equation*** can be rearranged to solve for the expected percentage appreciation or depreciation of the foreign currency:

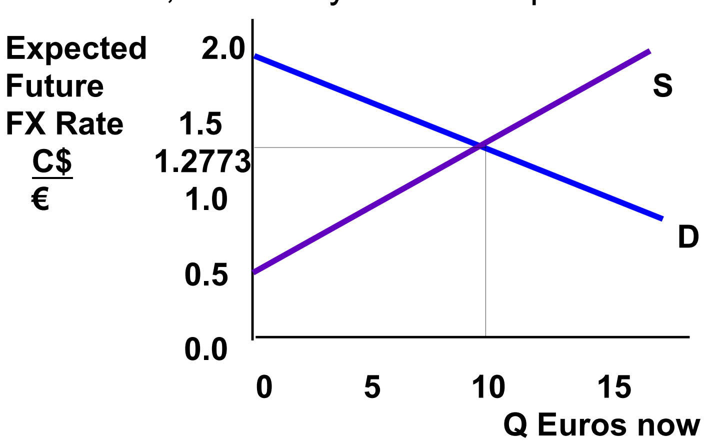
$$\frac{Ep_1}{p_0} = (1+r_P) = \frac{(1+r_D)}{(1+r_F)} \quad \text{so: } \frac{\$1.2773}{\$1.3021} = \frac{1.03}{1.05} = .980952 \quad \text{meaning:}$$

$Ep_1/p_0 = (1+r_P)$ where *r_P is the expected % appreciation or depreciation of the currency*.

Therefore $r_P = Ep_1/p_0 - 1 = (.980952-1) = -.0190476$ a little under 2%, which is very close to the interest rate differential between France and Canada.
Why isn't it exactly 2%? Because if the Euro depreciates you will lose not only on the original €767,990 you bought but also on the interest earned.

$(1+r_D) = (1+r_F)(Ep_1/p_0)$ or $(1+r_D) = (1+r_F)(1+r_P)$ solves to: 1.03 = 1.05(1-.0190476)

Uncovered interest parity tells us that if the € depreciates by 2% over the next year, then an investor would be indifferent as to whether she puts her $1 million in Canada or France, as each yields the equivalent rate of 3% in Canadian dollars.



The equilibrium future FX rate is C$/€ 1.2773 provided i_D of 3%, i_F of 5% and p_0 of 1.3021 are also equilibrium values. It means $1.2773 is what the market expects the Euro to be worth in one year.

The exchange rate of C$/€ 1.2773 was arrived at through the uncovered interest rate parity equation $Ep_1 = \frac{(1+r_D)}{(1+r_F)} \times p_0$.

***SPECULATIVE* STRATEGY *WHEN p_1 UNKNOWN*:**

If an investor is confident the Euro will depreciate by more than 2%, say 2.9%, from 1.3021 to ***1.2721***, the *speculative* strategy would be to borrow in France and invest in Canada. This is risky (speculative) because the investor does not know with certainty how much the Euro will really be worth in one year.

Today: Borrow the equivalent of $1 million in France at 5%.
$1/p_0$ = € 1/$1.3021= €.76799/C$ so borrow €767,990.
In one year you will owe €767,990*1.05 = €806,390.
Convert €767,990 back into C$1 million today and invest in Canada at 3%.

Next Year: Receive $1,030,000 from your Canadian investment. Pay off your Euro loan. Given your forecasted future FX rate (Ep_1) of C$***1.2721/***€, you expect the Euro loan to cost €806,390*1.2721=$1,025,808.

Expected Profit (π): $1,030,000 - $1,025,809 = **$4,191**

Risk: *If the Euro depreciates by less than 2% or even appreciates, the profits on this strategy will be less, or there may even be a loss.*

Compare $\frac{1}{p_0}(1+r_F)Ep_1$ to $(1+r_D)$ profit is the difference between what you would earn abroad vs at home.

$\frac{Ep_1}{p_0}(1+r_F)$ vs. $(1+r_D)$ Same ***arbitrage equation*** with terms rearranged.

$\frac{1.2721}{1.3021}(1.05) = 1.0258 < 1.03$ In this case the domestic return is higher.
A depositor would deposit in Canada.
A borrower would borrow in France.

That's why the arbitrage strategy is to borrow in France and invest in Canada, provided the investor is confident about the expected future C$/€ rate.

Suppose on the other hand you thought the Euro would only depreciate by 1.3% to 1.289, while interest rates stay same as above. Then the arbitrage equation becomes:

$\frac{1.289}{1.3021}(1.05) = 1.0394 > 1.03$ then the speculative strategy would be to borrow in Canada & invest in France.

Today: borrow $1 million in Canada at 3%, and owe 1,030,000 in one year.
Buys Euros with the C$1 million (€767,990) and invest at 5% in France, so that in one year you will have €767,990x1.05 = € 806,390.

In one year: translate the Euros back into Canadian dollars
€ 806,390x1.289 = C$1,039,437 and pay off C$ loan of $1,030,000, for a profit of C$1,039,437-C$1,030,000 = C$9,437.

CURRENCY FORWARDS & ARBITRAGE:

There are forward and futures contracts traded on currencies. These contracts allow you to buy or sell later at a price that is agreed to now.

The forward exchange rate available today is denoted as "f_0 " and it is arrived at through the following equation:

***Covered Interest rate parity equation (f_0 known)*:** $(1+r_D) = (1+r_F)(1/p_0)f_0$ or:
Equivalently, this can be expressed as:

$$\frac{f_0}{p_0} = \frac{(1+r_D)}{(1+r_F)} \quad \text{or} \quad f_0 = \frac{(1+r_D)p_0}{(1+r_F)}$$

The word "covered" reflects the fact that the currency to be bought or sold in a year has already been secured at a known forward rate today, and is therefore "covered".

Example: Suppose Seagrams has to buy wine from France in 1 year. It can contract today to buy € at the forward exchange rate f_0 for protection against fluctuations.

Using our same example, if the forward rate indicates the € will depreciate 2% over the next year, we would say 2% is ***the one-year forward "discount" of the €***.

What if the one-year forward price is temporarily out of equilibrium at $1.2934 instead of $1.2773? Seagrams can employ an arbitrage strategy (not speculating).

$\frac{1.2934}{1.3021}(1.05) = \mathbf{1.04298 > 1.03}$ In this case the domestic return is lower.
A depositor would deposit in France.
A borrower would borrow in Canada.

In this case the arbitrage strategy would be to borrow in Canada and invest in France.

ARBITRAGE STRATEGY WHEN f_0 IS KNOWN (*and different from Ep_1*) :

Today: Borrow $1 million in Canada at 3%. You will owe $1,030,000 next year. With the C$1 million, buy €767,990 worth of French government bonds paying 5% and receive €806,390 in one year. Sell €806,390 forward at the one-year forward price of ***$1.2934*** so that in a year you get €806,390 x $1.2934 = $1,042,985.

Next Year: Receive $1,042,985 - the proceeds of your forward sale of Euros.
Pay the $1,030,000 on your Canadian loan.
Arbitrage profit = $1,042,985 - $1,030,000 = **$12,985**.

The catch is that in an efficient market, arbitrage profits like this do not exist as financial institutions actually set their forward rates according to the covered interest arbitrage formula $f_0 = p_0 (1+r_D)/(1+r_F)$

PRACTICE PROBLEMS: You must know the following formulas:

$$Ep_1 = \frac{(1+r_D)}{(1+r_F)} \times p_0$$

This is the ***future exchange rate that is expected by the investor*** and is the rate that would make the investor indifferent between investing in the home country or the foreign country for a year. It is based on current known interest rates and the spot exchange rate.

$r_P = Ep_1/p_0 - 1$ This is the expected % appreciation or depreciation in the foreign currency and the answer should be similar to, *but not exactly the same as*, the domestic minus the foreign interest rate ($r_D - r_F$).

$$f_0 = \frac{(1+r_D)}{(1+r_F)} \times p_0$$

This is ***the actual one year forward exchange rate*** available in the market today. If you are given a figure for this in a problem and it differs from the expected rate Ep_1 above, then you are to assume that f_0 is temporarily out of equilibrium and arbitrage profits are possible.

Arbitrage equation when f_0 known: $\frac{f_0}{p_0}(1+r_F)$ vs. $(1+r_D)$ This compares what you can earn in foreign country to what you can earn at home.

Problem 1:

The one-year interest rate is 3.5% in Canada and 2.5% in Australia.
The spot price of one Australian dollar today (p_0) is C$1.210.
The known one-year forward price of an Australian dollar today (f_0) is C$1.225.

a) What is the expected value of the Australian dollar in one year?
What is the expected % appreciation or depreciation of the Australian dollar?

b) What arbitrage profits are possible with an investment of C$1 million?

Solution:

a) $\frac{1.035}{1.025} \times 1.210 = \mathbf{1.2218} = \mathbf{Ep_1}$ This is the expected value of the A$ in one year.

$\frac{1.2218}{1.210} - 1 = \mathbf{0.00975}$ Australian dollar is expected to appreciate by 0.975%.

b) $\frac{1.225}{1.210} \times 1.025 = \mathbf{1.03771 > 1.035}$ so borrow in Canada and invest in Australia.

Strategy Borrow C$1 million at 3.5% and owe C$1,035,000 in one year.
Use the C$1 million to buy Australian dollars: $1/p_0$ = 0.826446 x C$1 million = A$826,446 and invest the Australian dollars at 2.5% to earn 1.025 x 826,446 = A$847,107 in one year.
Sell A$847,107 forward at the current forward exchange rate of 1.225 so that you will receive C$1,037,707 in one year.

Next year: Take the C$1,037,707 received from the forward contract which has now matured and pay off the loan of C$1,035,000.
Arbitrage profit = 1,037,707-1,035,000 = **C$2,707.**

Problem 2:

The one-year interest rate is 3.95% in Canada and 3.25% in Italy.
The spot price of one Euro today is C$1.395.
The known one-year forward price of a Euro today is C$1.385.

a) What is the expected value of the Euro in one year?
What is the expected % appreciation or depreciation of the Euro?

b) What arbitrage profits are possible with an investment of C$1 million?

Solution:

a) $\frac{1.0395}{1.0325} \times 1.395 = \mathbf{1.4045} = Ep_1$

$\frac{1.4045}{1.395} - 1 = \mathbf{0.00681}$ Euro is expected to appreciate by 0.68%.

b) $\frac{1.385}{1.395} \times 1.0325 = \mathbf{1.0251 < 1.0395}$ so borrow in Italy and invest in Canada.

Strategy today: Borrow C$1 million worth of Euros today at 1/1.395 x C$1 mill. = €716,846 at 3.25% and owe €740,144 in one year (716,846x1.0325 = 740,144).

Use the €716,846 that you just borrowed to buy Canadian dollars today 716,846x1.395 = C$1 million and earn 3.95% on it in Canada. In one year you will have C$1 million x 1.0395 = C$1,039,500.

Sell C$1,039,500. forward at the current forward exchange rate of $1/f_0$ 0.72202 so that you will receive €750,542 in one year.

Next year: Take the €750,542 received from the forward contract which has now matured and pay off the loan of €740,144.

Arbitrage profit = 750,542 - 740,144 = €**10,398** x 1.4045 = **C$14,604**

Note: We used Ep_1 for the final calculation because problem assumes that $\boldsymbol{f_0}$ was temporarily out of equilibrium initially, and Ep_1 is correct.

Problem 3:

The one-year interest rate is 4.5% in Canada and 3.75% in the U.S.
The spot price of one U.S. dollar today is C$0.985.
The known one-year forward price of a U.S. dollar today is C$1.025.

a) What is the expected value of the U.S. dollar in one year?
What is the expected % appreciation or depreciation of the U.S. dollar?

b) What arbitrage profits are possible with an investment of C$1 million?

Solution:

a) $\frac{1.045}{1.0375} \times 0.985 = \mathbf{0.99212} = Ep_1$

$\frac{0.99212}{0.985} - 1 = \mathbf{0.00723}$ The U.S. dollar is expected to appreciate by 0.723%

b) $\frac{1.025}{0.985} \times 1.0375 = \mathbf{1.07963 > 1.045}$ so borrow in Canada and invest in the U.S.

Strategy today: Borrow C$1 million at 4.5% and owe C$1,045,000 in one year.

Use the C$1 million to buy U.S. dollars: $1/p_0$ x C$1 million :
$1/p_0$ = 1.015228 x C$1 million = US$1,015,228 and invest the U.S. dollars at 3.75% to earn 1.0375 x 1,015,228 = US$1,053,300 in one year.

Sell US$1,053,300 forward at the current forward rate of 1.025 so that you will receive C$1,079,632 in one year.

Next year: Take the C$1,079,632 received from the forward contract which has now matured and pay off the loan of C$1,045,000.

Arbitrage profit = 1,079,632 - 1,045,000 = **C$34,632.**

See next page for problem 4.

Problem 4:

The one-year interest rate is 3.15% in Canada and 3.75% in Italy.
The spot price of one Euro today is C$1.365.
The known one-year forward price of a Euro today is C$1.345.

a) What is the expected value of the Euro in one year?
What is the expected % appreciation or depreciation of the Euro?

b) What arbitrage profits are possible with an investment of C$500,000?

Solution:

a) $\frac{1.0315}{1.0375} \times 1.365 = \mathbf{1.3571} = Ep_1$

$\frac{1.3571}{1.365} - 1 = \mathbf{-0.00579}$ Euro is expected to depreciate by 0.579%.

b) $\frac{1.345}{1.365} \times 1.0375 = \mathbf{1.0223 < 1.0315}$ so borrow in Italy and invest in Canada.

Strategy today: Borrow C$500,000 worth of Euros today at 1/1.365. = €366,300 at 3.75% and owe €380,037 in one year (366,300 x1.0375 =€380,037).

Use the €366,300 that you just borrowed to buy Canadian dollars today 366,300 x1.365 = C$500,000 and earn 3.15% on it in Canada.
In one year you will have C$500,000 x 1.0315 = C$515,750.

Sell C$515,750 forward at the current forward exchange rate of $1/f_0$ = 0.74349 so that you will receive €383,457 in one year.

Next year: Take the €383,457 received from the forward contract which has now matured and pay off the loan of €380,037.

Arbitrage profit = 383,457 - 380,037 = €3,**420** x 1.3571 = **C$4,641**

Note: We used Ep_1 for the final calculation because problem assumes that $\boldsymbol{f_0}$ was temporarily out of equilibrium initially, and Ep_1 is correct.